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Oscillatory Motion

CHAPTER OUTLINE

- 15.1 Motion of an Object Attached to a Spring
- 15.2 Mathematical Representation of Simple Harmonic Motion
- 15.3 Energy of the Simple Harmonic Oscillator
- 15.4 Comparing Simple Harmonic Motion with Uniform Circular Motion
- 15.5 The Pendulum
- 15.6 Damped Oscillations
- 15.7 Forced Oscillations

ANSWERS TO QUESTIONS

- Q15.1** Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.
- Q15.2** You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.
- Q15.3** The two will be equal if and only if the position of the particle at time zero is its equilibrium position, which we choose as the origin of coordinates.

- Q15.4** (a) In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.
- (b) Velocity and acceleration are in the same direction half the time.
- (c) Acceleration is always opposite to the position vector, and never in the same direction.
- Q15.5** No. It is necessary to know both the position and velocity at time zero.
- Q15.6** The motion will still be simple harmonic motion, but the period of oscillation will be a bit larger. The effective mass of the system in $\omega = \left(\frac{k}{m_{\text{eff}}} \right)^{1/2}$ will need to include a certain fraction of the mass of the spring.

Q15.7 We assume that the coils of the spring do not hit one another. The frequency will be higher than f by the factor $\sqrt{2}$. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring, because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

Q15.8 No; Kinetic, Yes; Potential, No. For constant amplitude, the total energy $\frac{1}{2}kA^2$ stays constant. The kinetic energy $\frac{1}{2}mv^2$ would increase for larger mass if the speed were constant, but here the greater mass causes a decrease in frequency and in the average and maximum speed, so that the kinetic and potential energies at every point are unchanged.

Q15.9 Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

<i>Equation</i>	<i>Information given by equation</i>
$x(t) = A \cos(\omega t + \phi)$	position as a function of time
$v(t) = -\omega A \sin(\omega t + \phi)$	velocity as a function of time
$v(x) = \pm \omega (A^2 - x^2)^{1/2}$	velocity as a function of position
$a(t) = -\omega^2 A \cos(\omega t + \phi)$	acceleration as a function of time
$a(t) = -\omega^2 x(t)$	acceleration as a function of position

The angular frequency ω appears in every equation. It is a good idea to figure out the value of angular frequency early in the solution to a problem about vibration, and to store it in calculator memory.

Q15.10 We have $T_i = \sqrt{\frac{L_i}{g}}$ and $T_f = \sqrt{\frac{L_f}{g}} = \sqrt{\frac{2L_i}{g}} = \sqrt{2}T_i$. The period gets larger by $\sqrt{2}$ times. Changing the mass has no effect on the period of a simple pendulum.

Q15.11 (a) Period decreases. (b) Period increases. (c) No change.

Q15.12 No, the equilibrium position of the pendulum will be shifted (angularly) towards the back of the car. The period of oscillation will increase slightly, since the restoring force (in the reference frame of the accelerating car) is reduced.

Q15.13 The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

Q15.14 Shorten the pendulum to decrease the period between ticks.

Q15.15 No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

- Q15.16** Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.
- Q15.17** The phase constant must be π rad.
- Q15.18** Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.
- Q15.19** The release of air from one side of the parachute can make the parachute turn in the opposite direction, causing it to release air from the opposite side. This behavior will result in a periodic driving force that can set the parachute into side-to-side oscillation. If the amplitude becomes large enough, the parachute will not supply the needed air resistance to slow the fall of the unfortunate skydiver.
- Q15.20** An imperceptibly slight breeze may be blowing past the leaves in tiny puffs. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.
- Q15.21** We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob hops back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

SOLUTIONS TO PROBLEMS

Section 15.1 Motion of an Object Attached to a Spring

- P15.1** (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the motion is periodic.
- (b) To determine the period, we use: $x = \frac{1}{2}gt^2$.
- The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.909 \text{ s}$
- This equals one-half the period, so $T = 2(0.909 \text{ s}) = \span style="border: 1px solid black; padding: 2px;">1.82 \text{ s}.$
- (c) No. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

Section 15.2 Mathematical Representation of Simple Harmonic Motion

P15.2 (a) $x = (5.00 \text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00 \text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0 \text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) $A = \boxed{5.00 \text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14 \text{ s}}$

P15.3 $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$
or $\boxed{f = 1.50 \text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

***P15.4** (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45 \text{ kg } 9.8 \text{ m/s}^2}{0.35 \text{ m}} = 12.6 \text{ N/m}$$

we take the x -axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0 \text{ cm}\cos\sqrt{\frac{12.6 \text{ kg}}{0.45 \text{ kg}\cdot\text{s}^2}} 84.4 \text{ s} = 18.0 \text{ cm}\cos 446.6 \text{ rad} = \boxed{15.8 \text{ cm}}$$

(d) Now $446.6 \text{ rad} = 71 \times 2\pi + 0.497 \text{ rad}$. In each cycle the object moves $4(18) = 72 \text{ cm}$, so it has moved $71(72 \text{ cm}) + (18 - 15.8) \text{ cm} = \boxed{51.1 \text{ m}}$.

(b) By the same steps, $k = \frac{0.44 \text{ kg } 9.8 \text{ m/s}^2}{0.355 \text{ m}} = 12.1 \text{ N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0 \text{ cm}\cos\sqrt{\frac{12.1}{0.44}} 84.4 = 18.0 \text{ cm}\cos 443.5 \text{ rad} = \boxed{-15.9 \text{ cm}}$$

(e) $443.5 \text{ rad} = 70(2\pi) + 3.62 \text{ rad}$

$$\text{Distance moved} = 70(72 \text{ cm}) + 18 + 15.9 \text{ cm} = \boxed{50.7 \text{ m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

- P15.5** (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A \sin \omega t$

and

$$v = v_i \cos \omega t$$

Since $f = 1.50$ Hz,

$$\omega = 2\pi f = 3.00\pi$$

Also, $A = 2.00$ cm, so that

$$x = (2.00 \text{ cm}) \sin 3.00\pi t$$

- (b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi \text{ cm/s} = \boxed{18.8 \text{ cm/s}}$

The particle has this speed at $t = 0$ and next at

$$t = \frac{T}{2} = \boxed{\frac{1}{3} \text{ s}}$$

- (c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2 \text{ cm/s}^2 = \boxed{178 \text{ cm/s}^2}$

This positive value of acceleration first occurs at

$$t = \frac{3}{4}T = \boxed{0.500 \text{ s}}$$

- (d) Since $T = \frac{2}{3} \text{ s}$ and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in $1.00 \text{ s} \left(= \frac{3}{2}T \right)$, the particle will travel $8.00 \text{ cm} + 4.00 \text{ cm} = \boxed{12.0 \text{ cm}}$.

- P15.6** The proposed solution

$$x(t) = x_i \cos \omega t + \left(\frac{v_i}{\omega} \right) \sin \omega t$$

implies velocity

$$v = \frac{dx}{dt} = -x_i \omega \sin \omega t + v_i \cos \omega t$$

and acceleration

$$a = \frac{dv}{dt} = -x_i \omega^2 \cos \omega t - v_i \omega \sin \omega t = -\omega^2 \left(x_i \cos \omega t + \left(\frac{v_i}{\omega} \right) \sin \omega t \right) = -\omega^2 x$$

- (a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω . At $t = 0$ the equations reduce to $x = x_i$ and $v = v_i$ so they satisfy all the requirements.

- (b)
$$v^2 - ax = (-x_i \omega \sin \omega t + v_i \cos \omega t)^2 - (-x_i \omega^2 \cos \omega t - v_i \omega \sin \omega t) \left(x_i \cos \omega t + \left(\frac{v_i}{\omega} \right) \sin \omega t \right)$$

$$v^2 - ax = x_i^2 \omega^2 \sin^2 \omega t - 2x_i v_i \omega \sin \omega t \cos \omega t + v_i^2 \cos^2 \omega t$$

$$+ x_i^2 \omega^2 \cos^2 \omega t + x_i v_i \omega \cos \omega t \sin \omega t + x_i v_i \omega \sin \omega t \cos \omega t + v_i^2 \sin^2 \omega t = x_i^2 \omega^2 + v_i^2$$

So this expression is constant in time. On one hand, it must keep its original value $v_i^2 - a_i x_i$.

On the other hand, if we evaluate it at a turning point where $v = 0$ and $x = A$, it is

$A^2 \omega^2 + 0^2 = A^2 \omega^2$. Thus it is proved.

- P15.7** (a) $T = \frac{12.0 \text{ s}}{5} = \boxed{2.40 \text{ s}}$

- (b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417 \text{ Hz}}$

- (c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62 \text{ rad/s}}$

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***P15.8** The mass of the cube is

$$m = \rho V = (2.7 \times 10^3 \text{ kg/m}^3)(0.015 \text{ m})^3 = 9.11 \times 10^{-3} \text{ kg}$$

The spring constant of the strip of steel is

$$k = \frac{F}{x} = \frac{14.3 \text{ N}}{0.0275 \text{ m}} = 52.0 \text{ N/m}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{52 \text{ kg}}{\text{s}^2 9.11 \times 10^{-3} \text{ kg}}} = \boxed{12.0 \text{ Hz}}$$

P15.9 $f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ or $T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}$

Solving for k ,

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (7.00 \text{ kg})}{(2.60 \text{ s})^2} = \boxed{40.9 \text{ N/m}}.$$

***P15.10** $x = A \cos \omega t$ $A = 0.05 \text{ m}$ $v = -A\omega \sin \omega t$ $a = -A\omega^2 \cos \omega t$

If $f = 3600 \text{ rev/min} = 60 \text{ Hz}$, then $\omega = 120\pi \text{ s}^{-1}$

$$v_{\max} = 0.05(120\pi) \text{ m/s} = \boxed{18.8 \text{ m/s}} \quad a_{\max} = 0.05(120\pi)^2 \text{ m/s}^2 = \boxed{7.11 \text{ km/s}^2}$$

P15.11 (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8.00 \text{ N/m}}{0.500 \text{ kg}}} = 4.00 \text{ s}^{-1}$ so position is given by $x = 10.0 \sin(4.00t) \text{ cm}$.

From this we find that $v = 40.0 \cos(4.00t) \text{ cm/s}$ $v_{\max} = \boxed{40.0 \text{ cm/s}}$

$$a = -160 \sin(4.00t) \text{ cm/s}^2 \quad a_{\max} = \boxed{160 \text{ cm/s}^2}.$$

(b) $t = \left(\frac{1}{4.00}\right) \sin^{-1}\left(\frac{x}{10.0}\right)$ and when $x = 6.00 \text{ cm}$, $t = 0.161 \text{ s}$.

We find $v = 40.0 \cos[4.00(0.161)] = \boxed{32.0 \text{ cm/s}}$

$$a = -160 \sin[4.00(0.161)] = \boxed{-96.0 \text{ cm/s}^2}.$$

(c) Using $t = \left(\frac{1}{4.00}\right) \sin^{-1}\left(\frac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00 \text{ cm}$, $t = 0.232 \text{ s}$.

Therefore, $\Delta t = \boxed{0.232 \text{ s}}.$

P15.12 $m = 1.00 \text{ kg}$, $k = 25.0 \text{ N/m}$, and $A = 3.00 \text{ cm}$. At $t = 0$, $x = -3.00 \text{ cm}$

$$(a) \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00 \text{ rad/s}$$

$$\text{so that,} \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = \boxed{1.26 \text{ s}}$$

$$(b) \quad v_{\max} = A\omega = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s}) = \boxed{0.150 \text{ m/s}}$$

$$a_{\max} = A\omega^2 = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s})^2 = \boxed{0.750 \text{ m/s}^2}$$

(c) Because $x = -3.00 \text{ cm}$ and $v = 0$ at $t = 0$, the required solution is $x = -A \cos \omega t$

$$\text{or} \quad \boxed{x = -3.00 \cos(5.00t) \text{ cm}}$$

$$v = \frac{dx}{dt} = \boxed{15.0 \sin(5.00t) \text{ cm/s}}$$

$$a = \frac{dv}{dt} = \boxed{75.0 \cos(5.00t) \text{ cm/s}^2}$$

P15.13 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

$$\text{and } v_{\max} = \omega A = (6.28/\text{s})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}.$$

P15.14 (a) $v_{\max} = \omega A$

$$A = \frac{v_{\max}}{\omega} = \boxed{\frac{v}{\omega}}$$

$$(b) \quad x = -A \sin \omega t = \boxed{-\left(\frac{v}{\omega}\right) \sin \omega t}$$

Section 15.3 Energy of the Simple Harmonic Oscillator

P15.15 (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K + U)_i = (K + U)_f \quad 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(6.50 \text{ N/m})(0.100 \text{ m})^2 = \frac{1}{2}m(0.300 \text{ m/s})^2 + \frac{1}{2}(6.50 \text{ N/m})(5.00 \times 10^{-2} \text{ m})^2$$

$$32.5 \text{ mJ} = \frac{1}{2}m(0.300 \text{ m/s})^2 + 8.12 \text{ mJ} \quad m = \frac{2(24.4 \text{ mJ})}{9.0 \times 10^{-2} \text{ m}^2/\text{s}^2} = \boxed{0.542 \text{ kg}}$$

$$(b) \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.50 \text{ N/m}}{0.542 \text{ kg}}} = 3.46 \text{ rad/s} \quad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{3.46 \text{ rad/s}} = \boxed{1.81 \text{ s}}$$

$$(c) \quad a_{\max} = A\omega^2 = 0.100 \text{ m}(3.46 \text{ rad/s})^2 = \boxed{1.20 \text{ m/s}^2}$$

P15.16 $m = 200 \text{ g}$, $T = 0.250 \text{ s}$, $E = 2.00 \text{ J}$; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1 \text{ rad/s}$

(a) $k = m\omega^2 = 0.200 \text{ kg}(25.1 \text{ rad/s})^2 = \boxed{126 \text{ N/m}}$

(b) $E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00)}{126}} = \boxed{0.178 \text{ m}}$

P15.17 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2: \quad v = x\sqrt{\frac{k}{m}} = (3.16 \times 10^{-2} \text{ m})\sqrt{\frac{5.00 \times 10^6}{10^3}} = \boxed{2.23 \text{ m/s}}$$

P15.18 (a) $E = \frac{kA^2}{2} = \frac{250 \text{ N/m}(3.50 \times 10^{-2} \text{ m})^2}{2} = \boxed{0.153 \text{ J}}$

(b) $v_{\max} = A\omega$ where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.500}} = 22.4 \text{ s}^{-1}$ $v_{\max} = \boxed{0.784 \text{ m/s}}$

(c) $a_{\max} = A\omega^2 = 3.50 \times 10^{-2} \text{ m}(22.4 \text{ s}^{-1})^2 = \boxed{17.5 \text{ m/s}^2}$

P15.19 (a) $E = \frac{1}{2}kA^2 = \frac{1}{2}(35.0 \text{ N/m})(4.00 \times 10^{-2} \text{ m})^2 = \boxed{28.0 \text{ mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$
 $|v| = \sqrt{\frac{35.0}{50.0 \times 10^{-3}}}\sqrt{(4.00 \times 10^{-2})^2 - (1.00 \times 10^{-2})^2} = \boxed{1.02 \text{ m/s}}$

(c) $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}(35.0)\left[(4.00 \times 10^{-2})^2 - (3.00 \times 10^{-2})^2\right] = \boxed{12.2 \text{ mJ}}$

(d) $\frac{1}{2}kx^2 = E - \frac{1}{2}mv^2 = \boxed{15.8 \text{ mJ}}$

P15.20 (a) $k = \frac{|F|}{x} = \frac{20.0 \text{ N}}{0.200 \text{ m}} = \boxed{100 \text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0} \text{ rad/s}$ so $f = \frac{\omega}{2\pi} = \boxed{1.13 \text{ Hz}}$

(c) $v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41 \text{ m/s}}$ at $x = 0$

(d) $a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0 \text{ m/s}^2}$ at $x = \pm A$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00 \text{ J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}}(0.200)^2 = \boxed{1.33 \text{ m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\frac{0.200}{3}\right) = \boxed{3.33 \text{ m/s}^2}$

P15.21 (a) $E = \frac{1}{2}kA^2$, so if $A' = 2A$, $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore E increases by factor of 4.

(b) $v_{\max} = \sqrt{\frac{k}{m}}A$, so if A is doubled, $v_{\max}$ is doubled.

(c) $a_{\max} = \frac{k}{m}A$, so if A is doubled, $a_{\max}$ also doubles.

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ is independent of A , so the period is unchanged.

***P15.22** (a) $y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$
 $-11 \text{ m} = 0 + 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$

$$t = \sqrt{\frac{22 \text{ m} \cdot \text{s}^2}{9.8 \text{ m}}} = 1.50 \text{ s}$$

- (b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$

$$0 + mgy + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$65 \text{ kg } 9.8 \text{ m/s}^2 \cdot 36 \text{ m} = \frac{1}{2}k(25 \text{ m})^2$$

$$k = 73.4 \text{ N/m}$$

- (c) The spring extension at equilibrium is $x = \frac{F}{k} = \frac{65 \text{ kg } 9.8 \text{ m/s}^2}{73.4 \text{ N/m}} = 8.68 \text{ m}$, so this point is $11 + 8.68 \text{ m} = 19.7 \text{ m}$ below the bridge and the amplitude of her oscillation is $36 - 19.7 = 16.3 \text{ m}$.

(d) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4 \text{ N/m}}{65 \text{ kg}}} = 1.06 \text{ rad/s}$

- (e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher when $x = -8.68 \text{ m}$:

$$\text{In } x = A \cos \omega t,$$

$$16.3 \text{ m} = 16.3 \text{ m} \cos 0$$

$$-8.68 \text{ m} = 16.3 \text{ m} \cos \left(1.06 \frac{t}{\text{s}} \right)$$

$$1.06 \frac{t}{\text{s}} = -122^\circ = -2.13 \text{ rad}$$

$$t = -2.01 \text{ s}$$

Then $+2.01 \text{ s}$ is the time over which the spring stretches.

(f) total time = $1.50 \text{ s} + 2.01 \text{ s} = 3.50 \text{ s}$

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P15.23 Model the oscillator as a block-spring system.

From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$$v_{\max} = \omega A \text{ and } v = \frac{\omega A}{2} \quad \text{so} \quad \left(\frac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$$

From this we find $x^2 = \frac{3}{4} A^2$ and $x = \frac{\sqrt{3}}{2} A = \boxed{\pm 2.60 \text{ cm}}$ where $A = 3.00 \text{ cm}$

P15.24 The potential energy is

$$U_s = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \cos^2(\omega t).$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2} kA^2 2 \cos(\omega t) [-\omega \sin(\omega t)] = -\frac{1}{2} kA^2 \omega \sin 2\omega t.$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2}, 2\omega t = 2\pi + \frac{\pi}{2}, \text{ or in general, } 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n.$$

$$\text{Then, } t = \frac{\pi}{4\omega} (4n+1) = \frac{\pi(4n+1)}{4(3.60 \text{ s}^{-1})}$$

For $n=0$, this gives $t = \boxed{0.218 \text{ s}}$ while $n=1$ gives $t = \boxed{1.09 \text{ s}}$.

All other values of n yield times outside the specified range.

$$(b) \quad \left| \frac{dU_s}{dt} \right|_{\max} = \frac{1}{2} kA^2 \omega = \frac{1}{2} (3.24 \text{ N/m}) (5.00 \times 10^{-2} \text{ m})^2 (3.60 \text{ s}^{-1}) = \boxed{14.6 \text{ mW}}$$

Section 15.4 Comparing Simple Harmonic Motion with Uniform Circular Motion

P15.25 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the bump projected in a plane perpendicular to the tire.

(b) Since the car is moving with speed $v = 3.00 \text{ m/s}$, and its radius is 0.300 m , we have:

$$\omega = \frac{3.00 \text{ m/s}}{0.300 \text{ m}} = 10.0 \text{ rad/s}.$$

Therefore, the period of the motion is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0 \text{ rad/s})} = \boxed{0.628 \text{ s}}.$$

- P15.26** The angle of the crank pin is $\theta = \omega t$.
Its x -coordinate is

$$x = A \cos \theta = A \cos \omega t$$

where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A \cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.

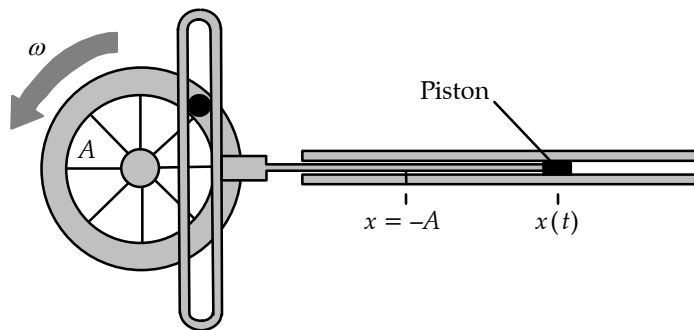


FIG. P15.26

Section 15.5 The Pendulum

P15.27 (a) $T = 2\pi \sqrt{\frac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80 \text{ m/s}^2)(12.0 \text{ s})^2}{4\pi^2} = \boxed{35.7 \text{ m}}$$

(b) $T_{\text{moon}} = 2\pi \sqrt{\frac{L}{g_{\text{moon}}}} = 2\pi \sqrt{\frac{35.7 \text{ m}}{1.67 \text{ m/s}^2}} = \boxed{29.1 \text{ s}}$

P15.28 The period in Tokyo is $T_T = 2\pi \sqrt{\frac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi \sqrt{\frac{L_C}{g_C}}$

We know $T_T = T_C = 2.00 \text{ s}$

For which, we see $\frac{L_T}{g_T} = \frac{L_C}{g_C}$

or $\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.9942}{0.9927} = \boxed{1.0015}$

P15.29 The swinging box is a physical pendulum with period $T = 2\pi \sqrt{\frac{I}{mgd}}$.

The moment of inertia is given approximately by

$$I = \frac{1}{3}mL^2 \text{ (treating the box as a rod suspended from one end).}$$

Then, with $L \approx 1.0 \text{ m}$ and $d \approx \frac{L}{2}$,

$$T \approx 2\pi \sqrt{\frac{\frac{1}{3}mL^2}{mg(\frac{L}{2})}} = 2\pi \sqrt{\frac{2L}{3g}} = 2\pi \sqrt{\frac{2(1.0 \text{ m})}{3(9.8 \text{ m/s}^2)}} = 1.6 \text{ s or } T \sim \boxed{10^0 \text{ s}}.$$

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P15.30 $\omega = \frac{2\pi}{T} :$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$

$\omega = \sqrt{\frac{g}{L}} :$ $L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$

P15.31 Using the simple harmonic motion model:

$$A = r\theta = 1 \text{ m} \cdot 15^\circ \frac{\pi}{180^\circ} = 0.262 \text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1 \text{ m}}} = 3.13 \text{ rad/s}$$

(a) $v_{\max} = A\omega = 0.262 \text{ m} \cdot 3.13/\text{s} = \boxed{0.820 \text{ m/s}}$

(b) $a_{\max} = A\omega^2 = 0.262 \text{ m} (3.13/\text{s})^2 = 2.57 \text{ m/s}^2$

$$a_{\tan} = r\alpha \quad \alpha = \frac{a_{\tan}}{r} = \frac{2.57 \text{ m/s}^2}{1 \text{ m}} = \boxed{2.57 \text{ rad/s}^2}$$

(c) $F = ma = 0.25 \text{ kg} \cdot 2.57 \text{ m/s}^2 = \boxed{0.641 \text{ N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv^2$ and $h = L(1 - \cos\theta)$
 $\therefore v_{\max} = \sqrt{2gL(1 - \cos\theta)} = \boxed{0.817 \text{ m/s}}$

(b) $I\alpha = mgL \sin\theta$
 $\alpha_{\max} = \frac{mgL \sin\theta}{mL^2} = \frac{g}{L} \sin\theta_i = \boxed{2.54 \text{ rad/s}^2}$

(c) $F_{\max} = mg \sin\theta_i = 0.250(9.80)(\sin 15.0^\circ) = \boxed{0.634 \text{ N}}$

P15.32 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field $(9.80 + 5.00) \text{ m/s}^2$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}}$$

$$T = \boxed{3.65 \text{ s}}$$

(b) $T = 2\pi \sqrt{\frac{5.00 \text{ m}}{(9.80 \text{ m/s}^2 - 5.00 \text{ m/s}^2)}} = \boxed{6.41 \text{ s}}$

(c) $g_{\text{eff}} = \sqrt{(9.80 \text{ m/s}^2)^2 + (5.00 \text{ m/s}^2)^2} = 11.0 \text{ m/s}^2$
 $T = 2\pi \sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = \boxed{4.24 \text{ s}}$

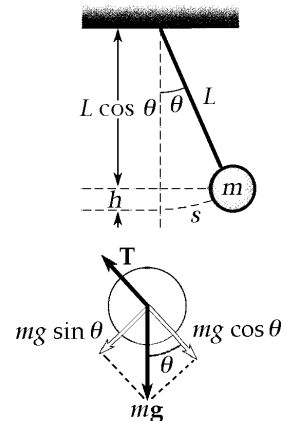


FIG. P15.31

P15.33 Referring to the sketch we have

$$F = -mg \sin \theta \quad \text{and} \quad \tan \theta = \frac{x}{R}$$

For small displacements, $\tan \theta \approx \sin \theta$

$$\text{and} \quad F = -\frac{mg}{R}x = -kx$$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}$.

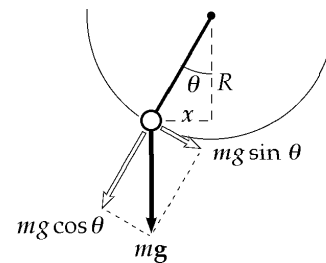


FIG. P15.33

P15.34 (a) $T = \frac{\text{total measured time}}{50}$

The measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

(b) $T = 2\pi\sqrt{\frac{L}{g}}$ so $g = \frac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
g (m/s^2)	9.91	9.87	9.76

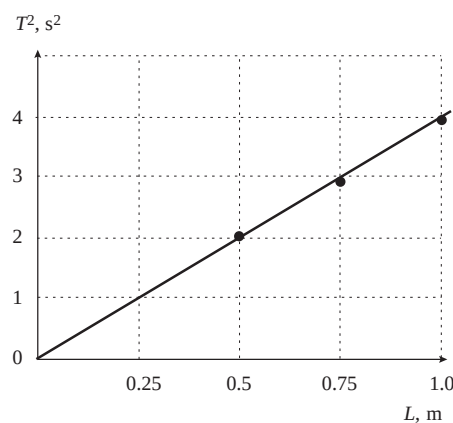


FIG. P15.34

Thus, $g_{\text{ave}} = \boxed{9.85 \text{ m/s}^2}$ this agrees with the accepted value of $g = 9.80 \text{ m/s}^2$ within 0.5%.

(c) From $T^2 = \left(\frac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph $= \frac{4\pi^2}{g} = 4.01 \text{ s}^2/\text{m}$.

Thus, $g = \frac{4\pi^2}{\text{slope}} = \boxed{9.85 \text{ m/s}^2}$. This is the same as the value in (b).

P15.35 $f = 0.450 \text{ Hz}$, $d = 0.350 \text{ m}$, and $m = 2.20 \text{ kg}$

$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \quad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2 \frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2 \frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2(0.450 \text{ s}^{-1})^2} = \boxed{0.944 \text{ kg} \cdot \text{m}^2}$$

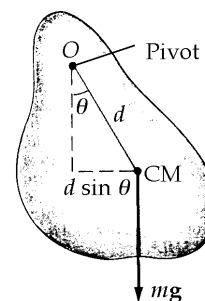


FIG. P15.35

P15.36 (a) The parallel-axis theorem:

$$\begin{aligned}
 I &= I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00 \text{ m})^2 + M(1.00 \text{ m})^2 \\
 &= M\left(\frac{13}{12} \text{ m}^2\right) \\
 T &= 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13 \text{ m}^2)}{12Mg(1.00 \text{ m})}} = 2\pi\sqrt{\frac{13 \text{ m}}{12(9.80 \text{ m/s}^2)}} = \boxed{2.09 \text{ s}}
 \end{aligned}$$

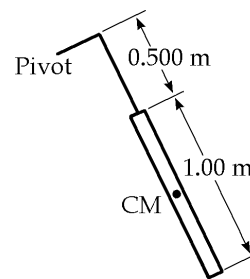


FIG. P15.36

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00 \text{ m}}{9.80 \text{ m/s}^2}} = 2.01 \text{ s} \quad \text{difference} = \frac{2.09 \text{ s} - 2.01 \text{ s}}{2.01 \text{ s}} = \boxed{4.08\%}$$

P15.37 (a) The parallel axis theorem says directly $I = I_{\text{CM}} + md^2$

$$\text{so} \quad T = 2\pi\sqrt{\frac{I}{mgd}} = \boxed{2\pi\sqrt{\frac{(I_{\text{CM}} + md^2)}{mgd}}}$$

(b) When d is very large $T \rightarrow 2\pi\sqrt{\frac{d}{g}}$ gets large.

When d is very small $T \rightarrow 2\pi\sqrt{\frac{I_{\text{CM}}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\begin{aligned}
 \frac{dT}{dd} = 0 &= \frac{d}{dd} 2\pi(I_{\text{CM}} + md^2)^{1/2} (mgd)^{-1/2} \\
 &= 2\pi(I_{\text{CM}} + md^2)^{1/2} \left(-\frac{1}{2}\right) (mgd)^{-3/2} mg + 2\pi(mgd)^{-1/2} \left(\frac{1}{2}\right) (I_{\text{CM}} + md^2)^{-1/2} 2md \\
 &= \frac{-\pi(I_{\text{CM}} + md^2)mg}{(I_{\text{CM}} + md^2)^{1/2} (mgd)^{3/2}} + \frac{2\pi md mgd}{(I_{\text{CM}} + md^2)^{1/2} (mgd)^{3/2}} = 0
 \end{aligned}$$

This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

$$\text{or} \quad \boxed{I_{\text{CM}} = md^2}.$$

P15.38 We suppose the stick moves in a horizontal plane. Then,

$$\begin{aligned}
 I &= \frac{1}{12}mL^2 = \frac{1}{12}(2.00 \text{ kg})(1.00 \text{ m})^2 = 0.167 \text{ kg} \cdot \text{m}^2 \\
 T &= 2\pi\sqrt{\frac{I}{\kappa}} \\
 \kappa &= \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(0.167 \text{ kg} \cdot \text{m}^2)}{(180 \text{ s})^2} = \boxed{203 \mu\text{N} \cdot \text{m}}
 \end{aligned}$$

P15.39 $T = 0.250 \text{ s}$, $I = mr^2 = (20.0 \times 10^{-3} \text{ kg})(5.00 \times 10^{-3} \text{ m})^2$

(a) $I = \boxed{5.00 \times 10^{-7} \text{ kg} \cdot \text{m}^2}$

(b) $I \frac{d^2\theta}{dt^2} = -\kappa\theta$; $\sqrt{\frac{\kappa}{I}} = \omega = \frac{2\pi}{T}$

$$\kappa = I\omega^2 = (5.00 \times 10^{-7}) \left(\frac{2\pi}{0.250} \right)^2 = \boxed{3.16 \times 10^{-4} \frac{\text{N} \cdot \text{m}}{\text{rad}}}$$

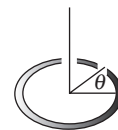


FIG. P15.39

Section 15.6 Damped Oscillations

P15.40 The total energy is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

Taking the time-derivative, $\frac{dE}{dt} = mv \frac{d^2x}{dt^2} + kxv$

Use Equation 15.31: $\frac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

Thus,

$$\boxed{\frac{dE}{dt} = -bv^2 < 0}$$

P15.41 $\theta_i = 15.0^\circ$

$$\theta(t = 1000) = 5.50^\circ$$

$$x = Ae^{-bt/2m}$$

$$\frac{x_{1000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1000)/2m}$$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1000)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00 \times 10^{-3} \text{ s}^{-1}}$$

P15.42 Show that $x = Ae^{-bt/2m} \cos(\omega t + \phi)$

is a solution of $-kx - b \frac{dx}{dt} = m \frac{d^2x}{dt^2}$ (1)

where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$. (2)

$$x = Ae^{-bt/2m} \cos(\omega t + \phi) \quad (3)$$

$$\frac{dx}{dt} = Ae^{-bt/2m} \left(-\frac{b}{2m} \right) \cos(\omega t + \phi) - Ae^{-bt/2m} \omega \sin(\omega t + \phi) \quad (4)$$

$$\begin{aligned} \frac{d^2x}{dt^2} = & -\frac{b}{2m} \left[Ae^{-bt/2m} \left(-\frac{b}{2m} \right) \cos(\omega t + \phi) - Ae^{-bt/2m} \omega \sin(\omega t + \phi) \right] \\ & - \left[Ae^{-bt/2m} \left(-\frac{b}{2m} \right) \omega \sin(\omega t + \phi) + Ae^{-bt/2m} \omega^2 \cos(\omega t + \phi) \right] \end{aligned} \quad (5)$$

continued on next page

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$\begin{aligned}
 & -kAe^{-bt/2m} \cos(\omega t + \phi) + \frac{b^2}{2m} Ae^{-bt/2m} \cos(\omega t + \phi) + b\omega Ae^{-bt/2m} \sin(\omega t + \phi) \\
 &= -\frac{b}{2} \left[Ae^{-bt/2m} \left(-\frac{b}{2m} \right) \cos(\omega t + \phi) - Ae^{-bt/2m} \omega \sin(\omega t + \phi) \right] \\
 &+ \frac{b}{2} Ae^{-bt/2m} \omega \sin(\omega t + \phi) - m\omega^2 Ae^{-bt/2m} \cos(\omega t + \phi)
 \end{aligned}$$

Compare the coefficients of $Ae^{-bt/2m} \cos(\omega t + \phi)$ and $Ae^{-bt/2m} \sin(\omega t + \phi)$:

$$\text{cosine-term: } -k + \frac{b^2}{2m} = -\frac{b}{2} \left(-\frac{b}{2m} \right) - m\omega^2 = \frac{b^2}{4m} - m \left(\frac{k}{m} - \frac{b^2}{4m^2} \right) = -k + \frac{b^2}{2m}$$

$$\text{sine-term: } b\omega = +\frac{b}{2}(\omega) + \frac{b}{2}(\omega) = b\omega$$

Since the coefficients are equal, $x = Ae^{-bt/2m} \cos(\omega t + \phi)$ is a solution of the equation.

***P15.43** The frequency if undamped would be $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.05 \times 10^4 \text{ N/m}}{10.6 \text{ kg}}} = 44.0/\text{s}.$

(a) With damping

$$\begin{aligned}
 \omega &= \sqrt{\omega_0^2 - \left(\frac{b}{2m} \right)^2} = \sqrt{\left(44 \frac{1}{\text{s}} \right)^2 - \left(\frac{3 \text{ kg}}{\text{s} \cdot 2 \cdot 10.6 \text{ kg}} \right)^2} \\
 &= \sqrt{1933.96 - 0.02} = 44.0 \frac{1}{\text{s}} \\
 f &= \frac{\omega}{2\pi} = \frac{44.0}{2\pi \text{ s}} = \boxed{7.00 \text{ Hz}}
 \end{aligned}$$

(b) In $x = A_0 e^{-bt/2m} \cos(\omega t + \phi)$ over one cycle, a time $T = \frac{2\pi}{\omega}$, the amplitude changes from A_0 to $A_0 e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0 - A_0 e^{-\pi b/m\omega}}{A_0} = 1 - e^{-\pi 3/(10.6 \cdot 44.0)} = 1 - e^{-0.0202} = 1 - 0.97998 = 0.0200 = \boxed{2.00\%}.$$

(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E = \frac{1}{2} k A^2 = \frac{1}{2} k A_0^2 e^{-2bt/2m} = E_0 e^{-bt/m}.$$

We specify

$$0.05E_0 = E_0 e^{-3t/10.6}$$

$$0.05 = e^{-3t/10.6}$$

$$e^{+3t/10.6} = 20$$

$$\frac{3t}{10.6} = \ln 20 = 3.00$$

$$t = \boxed{10.6 \text{ s}}$$

Section 15.7 Forced Oscillations

P15.44 (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4.30 \times 10^3 \text{ N/m}}{12.5 \text{ kg}}} = \boxed{2.95 \text{ Hz}}.$$

(b) From $x = A \cos \omega t$, $v = \frac{dx}{dt} = -A\omega \sin \omega t$, and $a = \frac{dv}{dt} = -A\omega^2 \cos \omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \quad \text{or} \quad A = \frac{g}{\omega^2} = \frac{g}{\frac{k}{m}} = \frac{gm}{k} \quad A = \frac{(9.80 \text{ m/s}^2)(12.5 \text{ kg})}{4.30 \times 10^3 \text{ N/m}} = \boxed{2.85 \text{ cm}}$$

P15.45 $F = 3.00 \cos(2\pi t) \text{ N}$ and $k = 20.0 \text{ N/m}$

(a) $\omega = \frac{2\pi}{T} = 2\pi \text{ rad/s}$ so $T = \boxed{1.00 \text{ s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16 \text{ rad/s}$

The equation for the amplitude of a driven oscillator,

with $b = 0$, gives $A = \left(\frac{F_0}{m} \right) (\omega^2 - \omega_0^2)^{-1} = \frac{3}{2} [4\pi^2 - (3.16)^2]^{-1}$

Thus $A = 0.0509 \text{ m} = \boxed{5.09 \text{ cm}}.$

P15.46 $F_0 \cos \omega t - kx = m \frac{d^2 x}{dt^2}$ $\omega_0 = \sqrt{\frac{k}{m}}$ (1)

$x = A \cos(\omega t + \phi)$ (2)

$\frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ (3)

$\frac{d^2 x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0 \cos \omega t - kA \cos(\omega t + \phi) = m(-A\omega^2) \cos(\omega t + \phi)$

Solve for the amplitude: $(kA - mA\omega^2) \cos(\omega t + \phi) = F_0 \cos \omega t$

These will be equal, provided only that ϕ must be zero and $kA - mA\omega^2 = F_0$

Thus, $A = \frac{\frac{F_0}{m}}{\left(\frac{k}{m}\right) - \omega^2}$

P15.47 From the equation for the amplitude of a driven oscillator with no damping,

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2}}$$

$$\omega = 2\pi f = (20.0\pi \text{ s}^{-1}) \qquad \omega_0^2 = \frac{k}{m} = \frac{200}{\left(\frac{40.0}{9.80}\right)} = 49.0 \text{ s}^{-2}$$

$$F_0 = mA(\omega^2 - \omega_0^2)$$

$$F_0 = \left(\frac{40.0}{9.80}\right)(2.00 \times 10^{-2})(3950 - 49.0) = \boxed{318 \text{ N}}$$

P15.48 $A = \frac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$

With $b = 0$, $A = \frac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \frac{F_{\text{ext}}/m}{\pm(\omega^2 - \omega_0^2)} = \pm \frac{F_{\text{ext}}/m}{\omega^2 - \omega_0^2}$

Thus, $\omega^2 = \omega_0^2 \pm \frac{F_{\text{ext}}/m}{A} = \frac{k}{m} \pm \frac{F_{\text{ext}}}{mA} = \frac{6.30 \text{ N/m}}{0.150 \text{ kg}} \pm \frac{1.70 \text{ N}}{(0.150 \text{ kg})(0.440 \text{ m})}$

This yields $\omega = 8.23 \text{ rad/s}$ or $\omega = 4.03 \text{ rad/s}$

Then, $f = \frac{\omega}{2\pi}$ gives either $f = \boxed{1.31 \text{ Hz}}$ or $f = \boxed{0.641 \text{ Hz}}$

P15.49 The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.80 \text{ m/s}^2}{0.0821 \text{ m}}} = \boxed{1.74 \text{ Hz}}.$$

***P15.50** For the resonance vibration with the occupants in the car, we have for the spring constant of the suspension

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \qquad k = 4\pi^2 f^2 m = 4\pi^2 (1.8 \text{ s}^{-1})^2 (1130 \text{ kg} + 4(72.4 \text{ kg})) = 1.82 \times 10^5 \text{ kg/s}^2$$

Now as the occupants exit $x = \frac{F}{k} = \frac{4(72.4 \text{ kg})(9.8 \text{ m/s}^2)}{1.82 \times 10^5 \text{ kg/s}^2} = \boxed{1.56 \times 10^{-2} \text{ m}}$

Additional Problems

P15.51 Let F represent the tension in the rod.

(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point P . Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}.$$

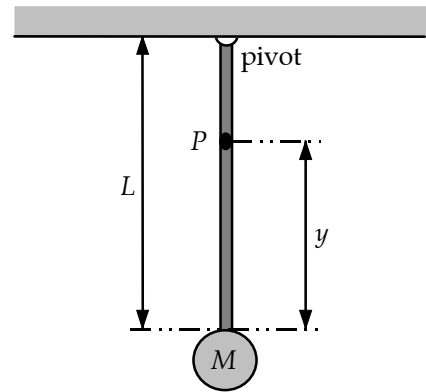


FIG. P15.51

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M\left(\frac{L}{2}\right) + ML}{M + M} = \frac{3L}{4} \text{ and } T = 2\pi\sqrt{\frac{\frac{4}{3}ML^2}{(2M)g\left(\frac{3L}{4}\right)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}.$$

For $L = 2.00 \text{ m}$, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}.$

P15.52 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100 \text{ N/m})(0.200 \text{ m})^2 = 2.00 \text{ J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0 \text{ kg})v^2 = (8.00 \text{ kg})v^2.$$

Therefore,

$$(8.00 \text{ kg})v^2 = 2.00 \text{ J, and } v = \boxed{0.500 \text{ m/s}}.$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

continued on next page

(b) The energy of the m_1 -spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00 \text{ kg})(0.500 \text{ m/s})^2 = 1.125 \text{ J}.$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1 -spring system. Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \text{ or } A' = 0.150 \text{ m}.$$

The period of the m_1 -spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885 \text{ s}$

and it takes $\frac{1}{4}T = 0.471 \text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500 \text{ m/s}(0.471 \text{ s}) - 0.150 \text{ m} = 0.0856 = \boxed{8.56 \text{ cm}}.$$

P15.53 $\left(\frac{d^2x}{dt^2}\right)_{\max} = A\omega^2$

$$f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$$

$$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62 \text{ cm}}$$

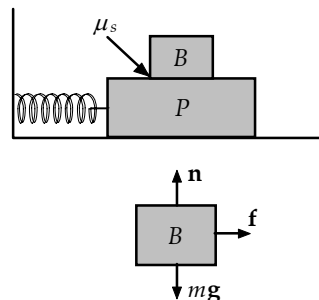


FIG. P15.53

P15.54 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m(4\pi^2 Af^2) \quad \text{or} \quad A = \frac{\mu_s g}{4\pi^2 f^2}.$$

P15.55 Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as D and to the diatomic molecule of hydrogen-1 as H .

$$M_D = 2M_H \quad \frac{\omega_D}{\omega_H} = \frac{\sqrt{\frac{k}{M_D}}}{\sqrt{\frac{k}{M_H}}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \quad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919 \times 10^{14} \text{ Hz}}$$

P15.56 The kinetic energy of the ball is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\Omega^2$, where Ω is the rotation rate of the ball about its center of mass. Since the center of the ball moves along a circle of radius $4R$, its displacement from equilibrium is $s = (4R)\theta$ and its speed is $v = \frac{ds}{dt} = 4R\left(\frac{d\theta}{dt}\right)$. Also, since the ball rolls without slipping,

$$v = \frac{ds}{dt} = R\Omega \quad \text{so} \quad \Omega = \frac{v}{R} = 4\left(\frac{d\theta}{dt}\right)$$

The kinetic energy is then

$$\begin{aligned} K &= \frac{1}{2}m\left(4R\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(4\frac{d\theta}{dt}\right)^2 \\ &= \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2 \end{aligned}$$

When the ball has an angular displacement θ , its center is distance $h = 4R(1 - \cos\theta)$ higher than when at the equilibrium position. Thus, the potential energy is $U_g = mgh = 4mgR(1 - \cos\theta)$. For small angles, $(1 - \cos\theta) \approx \frac{\theta^2}{2}$ (see Appendix B). Hence, $U_g \approx 2mgR\theta^2$, and the total energy is

$$E = K + U_g = \frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2 + 2mgR\theta^2.$$

Since $E = \text{constant}$ in time, $\frac{dE}{dt} = 0 = \frac{112mR^2}{5}\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2} + 4mgR\theta\left(\frac{d\theta}{dt}\right)$.

This reduces to $\frac{28R}{5}\frac{d^2\theta}{dt^2} + g\theta = 0$, or $\frac{d^2\theta}{dt^2} = -\left(\frac{5g}{28R}\right)\theta$.

With the angular acceleration equal to a negative constant times the angular position, this is in the defining form of a simple harmonic motion equation with $\omega = \sqrt{\frac{5g}{28R}}$.

The period of the simple harmonic motion is then $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{28R}{5g}}}$.

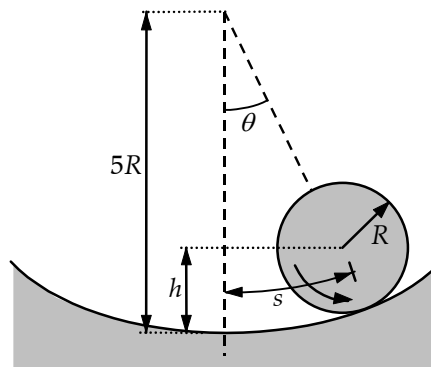


FIG. P15.56

P15.57 (a)

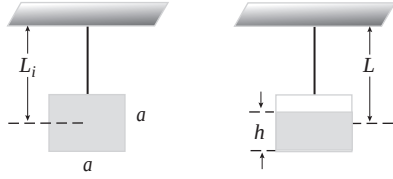


FIG. P15.57(a)

$$(b) \quad T = 2\pi \sqrt{\frac{L}{g}} \quad \frac{dT}{dt} = \frac{\pi}{\sqrt{g}} \frac{1}{\sqrt{L}} \frac{dL}{dt} \quad (1)$$

We need to find $L(t)$ and $\frac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2}; \quad \frac{dL}{dt} = -\left(\frac{1}{2}\right) \frac{dh}{dt}.$$

But $\frac{dM}{dt} = \rho \frac{dV}{dt} = -\rho A \frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A} \frac{dM}{dt}; \quad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right) \frac{dM}{dt} \quad (2)$$

$$\text{Also, } \int_{L_i}^L dL = \left(\frac{1}{2\rho A}\right) \left(\frac{dM}{dt}\right) t = L - L_i \quad (3)$$

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \left[\frac{\pi}{\sqrt{g}} \left(\frac{1}{2\rho a^2} \right) \left(\frac{dM}{dt} \right) \frac{1}{\sqrt{L_i + \frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right) t}} \right].$$

(c) Substitute Equation (3) into the equation for the period.

$$T = \frac{2\pi}{\sqrt{g}} \sqrt{L_i + \frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right) t}$$

Or one can obtain T by integrating (b):

$$\begin{aligned} \int_T^{T_i} dT &= \frac{\pi}{\sqrt{g}} \left(\frac{1}{2\rho a^2} \right) \left(\frac{dM}{dt} \right) \int_0^t \frac{dt}{\sqrt{L_i + \frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right) t}} \\ T - T_i &= \frac{\pi}{\sqrt{g}} \left(\frac{1}{2\rho a^2} \right) \left(\frac{dM}{dt} \right) \left[\frac{2}{\frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right)} \right] \left[\sqrt{L_i + \frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right) t} - \sqrt{L_i} \right] \end{aligned}$$

$$\text{But } T_i = 2\pi \sqrt{\frac{L_i}{g}}, \text{ so } T = \frac{2\pi}{\sqrt{g}} \sqrt{L_i + \frac{1}{2\rho a^2} \left(\frac{dM}{dt} \right) t}.$$

P15.58 $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\frac{4\pi^2 m}{T^2}}$

(b) $m' = \frac{k(T')^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

P15.59 We draw a free-body diagram of the pendulum. The force $\mathbf{H}$ exerted by the hinge causes no torque about the axis of rotation.

$$\tau = I\alpha \quad \text{and} \quad \frac{d^2\theta}{dt^2} = -\alpha$$

$$\tau = MgL \sin\theta + kxh \cos\theta = -I \frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

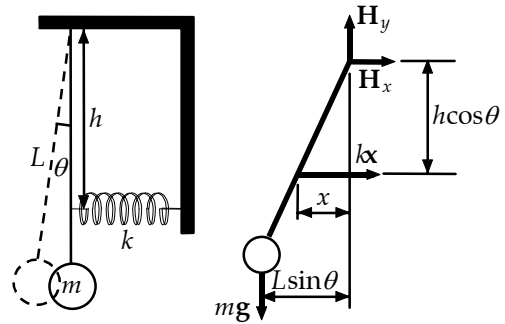


FIG. P15.59

Therefore, $\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$

$$\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi} \sqrt{\frac{MgL + kh^2}{ML^2}}}$$

***P15.60** (a) In $x = A \cos(\omega t + \phi)$,
we have at $t = 0$
This requires $\phi = 90^\circ$, so
And this is equivalent to
Numerically we have
and $v_{\max} = \omega A$
So

$$v = -\omega A \sin(\omega t + \phi)$$

$$v = -\omega A \sin\phi = -v_{\max}$$

$$x = A \cos(\omega t + 90^\circ)$$

$$x = -A \sin\omega t$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50 \text{ N/m}}{0.5 \text{ kg}}} = 10 \text{ s}^{-1}$$

$$20 \text{ m/s} = (10 \text{ s}^{-1})A \quad A = 2 \text{ m}$$

$$\boxed{x = (-2 \text{ m}) \sin[(10 \text{ s}^{-1})t]}$$

(b) In $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$,

implies

$$\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$$

$$\frac{1}{3} \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \quad \frac{4}{3}x^2 = A^2$$

$$x = \pm \sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73 \text{ m}}$$

continued on next page

$$(c) \quad \omega = \sqrt{\frac{g}{L}} \quad L = \frac{g}{\omega^2} = \frac{9.8 \text{ m/s}^2}{(10 \text{ s}^{-1})^2} = \boxed{0.0980 \text{ m}}$$

(d) In $x = (-2 \text{ m}) \sin[(10 \text{ s}^{-1})t]$
the particle is at $x = 0$ at $t = 0$, at $10t = \pi \text{ s}$, and so on.

The particle is at $x = 1 \text{ m}$

when $-\frac{1}{2} = \sin[(10 \text{ s}^{-1})t]$

with solutions $(10 \text{ s}^{-1})t = -\frac{\pi}{6}$

$$(10 \text{ s}^{-1})t = \pi + \frac{\pi}{6}, \text{ and so on.}$$

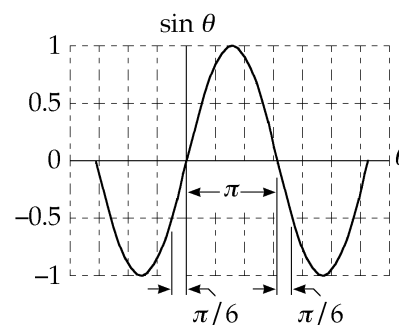


FIG. P15.60(d)

The minimum time for the motion is Δt in $10\Delta t = \left(\frac{\pi}{6}\right) \text{ s}$

$$\Delta t = \left(\frac{\pi}{60}\right) \text{ s} = \boxed{0.0524 \text{ s}}$$

P15.61 (a) At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0L$$

where x_0 is the equilibrium compression.

After displacement by a small angle,

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k(x_0 - L\theta)L = -k\theta L^2$$

But, $\sum \tau = I\alpha = \frac{1}{3}mL^2 \frac{d^2\theta}{dt^2}$. So $\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$.

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\omega^2 = \frac{3k}{m}$.

$$(b) \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{3k}{m}} = \frac{1}{2\pi} \sqrt{\frac{3(100 \text{ N/m})}{5.00 \text{ kg}}} = \boxed{1.23 \text{ Hz}}$$

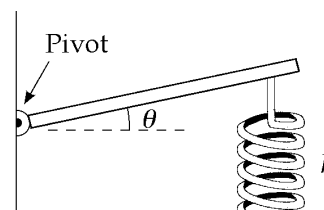


FIG. P15.61

***P15.62** As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A = \sqrt{\frac{100 \text{ N/m}}{4 \text{ kg}}} 2 \text{ m} = 10.0 \text{ m/s}.$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4 \text{ kg}(10 \text{ m/s}) + (6 \text{ kg})0 = (10 \text{ kg})v_{\max}$$

$$v_{\max} = 4.00 \text{ m/s}$$

(a) The new amplitude is given by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$10 \text{ kg}(4 \text{ m/s})^2 = (100 \text{ N/m})A^2$$

$$A = 1.26 \text{ m}$$

Thus the amplitude has decreased by $2.00 \text{ m} - 1.26 \text{ m} = \boxed{0.735 \text{ m}}$

(b) The old period was

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4 \text{ kg}}{100 \text{ N/m}}} = 1.26 \text{ s}$$

The new period is

$$T = 2\pi\sqrt{\frac{10}{100}} \text{ s} = 1.99 \text{ s}$$

The period has increased by $1.99 \text{ s} - 1.26 \text{ s} = \boxed{0.730 \text{ s}}$

(c) The old energy was

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(4 \text{ kg})(10 \text{ m/s})^2 = 200 \text{ J}$$

The new mechanical energy is

$$\frac{1}{2}(10 \text{ kg})(4 \text{ m/s})^2 = 80 \text{ J}$$

The energy has decreased by 120 J.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

P15.63 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00 \text{ s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3 \text{ J}}$

(c) At maximum angular displacement

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g} = 0.217 \text{ m}$$

$$h = L - L \cos \theta = L(1 - \cos \theta)$$

$$\cos \theta = 1 - \frac{h}{L}$$
 $\theta = 25.5^\circ$

464 Oscillatory Motion

P15.64 One can write the following equations of motion:

$$T - kx = 0 \quad (\text{describes the spring})$$

$$mg - T' = ma = m \frac{d^2x}{dt^2} \quad (\text{for the hanging object})$$

$$R(T' - T) = I \frac{d^2\theta}{dt^2} = \frac{I}{R} \frac{d^2x}{dt^2} \quad (\text{for the pulley})$$

$$\text{with } I = \frac{1}{2}MR^2$$

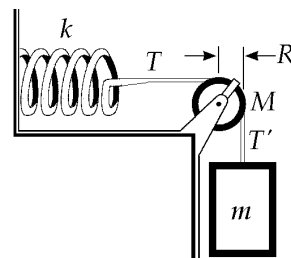


FIG. P15.64

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right) \frac{d^2x}{dt^2} + kx = mg.$$

The solution is $x(t) = A \sin \omega t + \frac{mg}{k}$ (where $\frac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi} \sqrt{\frac{100 \text{ N/m}}{0.200 \text{ kg} + \frac{1}{2}M}}.$$

(a) For $M = 0$ $f = \boxed{3.56 \text{ Hz}}$

(b) For $M = 0.250 \text{ kg}$ $f = \boxed{2.79 \text{ Hz}}$

(c) For $M = 0.750 \text{ kg}$ $f = \boxed{2.10 \text{ Hz}}$

P15.65 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then, $k = \frac{F}{x} = \frac{980 \text{ N}}{2.00 \times 10^{-2} \text{ m}} = 4.90 \times 10^4 \text{ N/m}$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4.90 \times 10^4 \text{ N/m}}{500 \text{ kg}}} = 1.58 \text{ Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0 \text{ m/s}}{1.58 \text{ s}^{-1}} = 12.7 \text{ m} \quad \boxed{\sim 10^1 \text{ m}}.$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

- P15.66** (a) For each segment of the spring

$$dK = \frac{1}{2}(dm)v_x^2.$$

Also, $v_x = \frac{x}{\ell}v$ and $dm = \frac{m}{\ell}dx$.

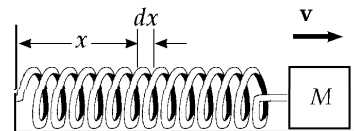


FIG. P15.66

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^\ell \left(\frac{x^2 v^2}{\ell^2} \right) \frac{m}{\ell} dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}.$$

(b) $\omega = \sqrt{\frac{k}{m_{\text{eff}}}}$ and $\frac{1}{2}m_{\text{eff}}v^2 = \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}}.$

- P15.67** (a) $\sum \mathbf{F} = -2T \sin \theta \hat{\mathbf{j}}$ where $\theta = \tan^{-1}\left(\frac{y}{L}\right)$

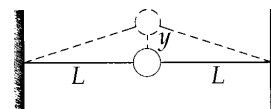


FIG. P15.67

Therefore, for a small displacement

$$\sin \theta \approx \tan \theta = \frac{y}{L} \quad \text{and} \quad \boxed{\sum \mathbf{F} = \frac{-2Ty}{L} \hat{\mathbf{j}}}$$

- (b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum \mathbf{F} = -k\mathbf{x} \quad \text{becomes here} \quad \sum \mathbf{F} = -\frac{2T}{L}\mathbf{y}.$$

Therefore, the effective spring constant is $\frac{2T}{L}$ and $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}}.$

P15.68 (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

and empirically

$$Mg = 1.74x - 0.113$$

so $k = \boxed{1.74 \text{ N/m} \pm 6\%}$.

$M, \text{ kg}$	$x, \text{ m}$	$Mg, \text{ N}$
0.020 0	0.17	0.196
0.040 0	0.293	0.392
0.050 0	0.353	0.49
0.060 0	0.413	0.588
0.070 0	0.471	0.686
0.080 0	0.493	0.784

(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.0589$$

so $k = \frac{4\pi^2}{21.7} = \boxed{1.82 \text{ N/m} \pm 3\%}$

Time, s	$T, \text{ s}$	$M, \text{ kg}$	$T^2, \text{ s}^2$
7.03	0.703	0.020 0	0.494
9.62	0.962	0.040 0	0.925
10.67	1.067	0.050 0	1.138
11.67	1.167	0.060 0	1.362
12.52	1.252	0.070 0	1.568
13.41	1.341	0.080 0	1.798

The k values $1.74 \text{ N/m} \pm 6\%$

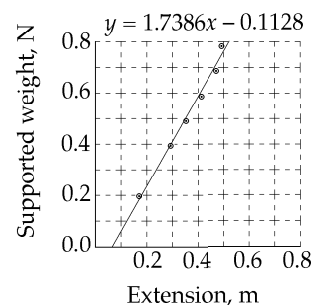
and $1.82 \text{ N/m} \pm 3\%$ differ by 4%

so $\boxed{\text{they agree.}}$

(c) Utilizing the axis-crossing point, $m_s = 3\left(\frac{0.0589}{21.7}\right) \text{ kg} = \boxed{8 \text{ grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

Static stretching of a spring



Squared period as a function of the mass of an object bouncing on a spring

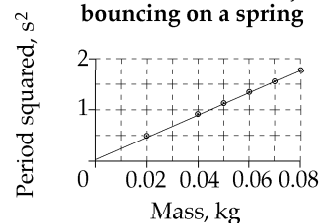


FIG. P15.68

P15.69 (a) $\Delta K + \Delta U = 0$
 Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$
 where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2}I\omega^2$, but

$$h = R - R\cos\theta = R(1 - \cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

Substituting we find

$$mgR(1 - \cos\theta) = \frac{1}{2} \left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2 \right) \frac{v^2}{R^2}$$

$$mgR(1 - \cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2} \right] v^2$$

and $v^2 = 4gR \frac{(1 - \cos\theta)}{\left(\frac{M}{m} + \frac{r^2}{R^2} + 2 \right)}$

so
$$v = 2 \sqrt{\frac{Rg(1 - \cos\theta)}{\frac{M}{m} + \frac{r^2}{R^2} + 2}}$$

(b) $T = 2\pi \sqrt{\frac{I}{m_T g d_{\text{CM}}}}$

$$m_T = m + M$$

$$d_{\text{CM}} = \frac{mR + M(0)}{m + M}$$

$$T = 2\pi \sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}$$

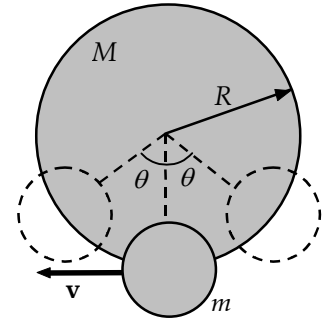


FIG. P15.69

P15.70 (a) We require $Ae^{-bt/2m} = \frac{A}{2}$ $e^{+bt/2m} = 2$

or $\frac{bt}{2m} = \ln 2$ or $\frac{0.100 \text{ kg/s}}{2(0.375 \text{ kg})}t = 0.693$ $\therefore t = \boxed{5.20 \text{ s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where $\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/2m}$. We require $\frac{1}{2}kA^2e^{-bt/2m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2$ $\therefore t = \frac{m \ln 2}{b} = \frac{0.375 \text{ kg}(0.693)}{0.100 \text{ kg/s}} = \boxed{2.60 \text{ s}}$

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{\frac{dE}{dt}}{E} = \frac{\frac{d}{dt}\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{\frac{dA}{dt}}{A}$$

two times faster than the fractional rate of change in amplitude.

- P15.71** (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2 .

By Newton's third law, we expect

$$k_1 x_1 = k_2 x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

we find

$$x_1 = \left[\frac{k_2}{k_1 + k_2} \right] x$$

The force on either spring is given by

$$F_1 = \left[\frac{k_1 k_2}{k_1 + k_2} \right] x = ma$$

where a is the acceleration of the mass m .

This is in the form

$$F = k_{\text{eff}} x = ma$$

and

$$T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}} = \boxed{2\pi \sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$$

- (b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1 + k_2)x \quad \text{and} \quad k_{\text{eff}} = k_1 + k_2$$

so that

$$T = \boxed{2\pi \sqrt{\frac{m}{k_1 + k_2}}}.$$

- P15.72** Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2 \ell g = 0.$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2 (\ell - x)g = Ma.$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 g x = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M} \right) x = -\omega^2 x$$

The opposite direction and direct proportionality of a to x imply SHM with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r} \right) \sqrt{\frac{\pi M}{\rho g}}}$$

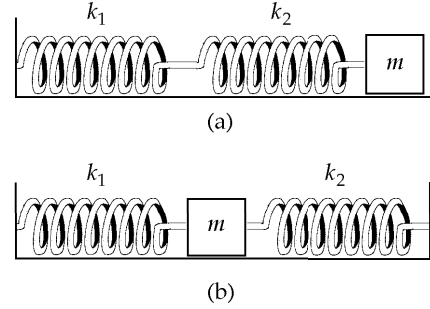


FIG. P15.71

P15.73 For $\theta_{\max} = 5.00^\circ$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{\max} \cos \omega t$. The period is $T = 2.20$ s.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. (°/s ²)	$\theta_{\max} \cos \omega t$
0.000	5.000 0	0.000 0	-40.781 5	5.000 0
0.004	4.999 3	-0.163 1	-40.776 2	4.999 7
0.008	4.998 0	-0.326 2	-40.765 6	4.998 7
...				
0.544	0.056 0	-14.282 3	-0.457 6	0.081 0
0.548	-0.001 1	-14.284 2	0.009 0	0.023 9
0.552	-0.058 2	-14.284 1	0.475 6	-0.033 3
...				
1.092	-4.999 4	-0.319 9	40.776 5	-4.998 9
1.096	-5.000 0	-0.156 8	40.781 6	-4.999 8
1.100	-5.000 0	0.006 3	40.781 4	-5.000 0
1.104	-4.999 3	0.169 4	40.775 9	-4.999 6
...				
1.644	-0.063 8	14.282 4	0.439 7	-0.071 6
1.648	0.003 3	14.284 2	-0.027 0	-0.014 5
1.652	0.060 4	14.284 1	-0.493 6	0.042 7
...				
2.192	4.999 4	0.313 7	-40.776 8	4.999 1
2.196	5.000 0	0.150 6	-40.781 7	4.999 9
2.200	5.000 0	-0.012 6	-40.781 3	5.000 0
2.204	4.999 3	-0.175 7	-40.775 6	4.999 4

For $\theta_{\max} = 100^\circ$, the simple harmonic motion approximation $\theta_{\max} \cos \omega t$ diverges greatly from the Euler calculation. The period is $T = 2.71$ s, larger than the small-angle period by 23%.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. (°/s ²)	$\theta_{\max} \cos \omega t$
0.000	100.000 0	0.000 0	-460.606 6	100.000 0
0.004	99.992 6	-1.843 2	-460.817 3	99.993 5
0.008	99.977 6	-3.686 5	-460.838 2	99.973 9
...				
1.096	-84.744 9	-120.191 0	465.948 8	-99.995 4
1.100	-85.218 2	-118.327 2	466.286 9	-99.999 8
1.104	-85.684 0	-116.462 0	466.588 6	-99.991 1
...				
1.348	-99.996 0	-3.053 3	460.812 5	-75.797 9
1.352	-100.000 8	-1.210 0	460.805 7	-75.047 4
1.356	-99.998 3	0.633 2	460.809 3	-74.287 0
...				
2.196	40.150 9	224.867 7	-301.713 2	99.997 1
2.200	41.045 5	223.660 9	-307.260 7	99.999 3
2.204	41.935 3	222.431 8	-312.703 5	99.988 5
...				
2.704	99.998 5	2.420 0	-460.809 0	12.642 2
2.708	100.000 8	0.576 8	-460.805 7	11.507 5
2.712	99.995 7	-1.266 4	-460.812 9	10.371 2

Motion of a Simple Pendulum

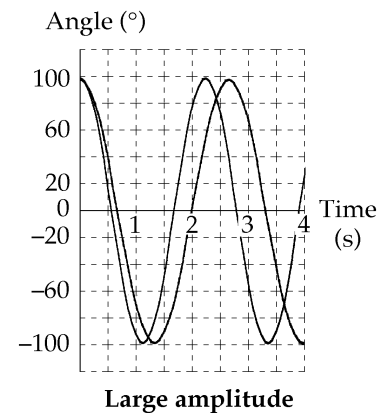
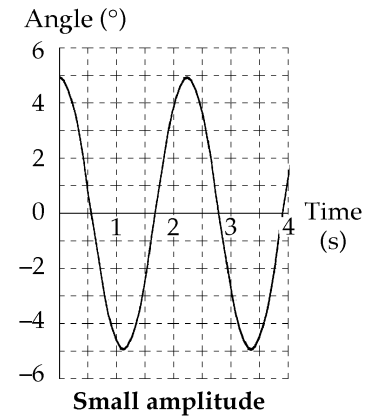


FIG. P15.73

- *P15.74** (a) The block moves with the board in what we take as the positive x direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction $\mu_s n = \mu_s mg$. This occurs at $x = \frac{\mu_s mg}{k}$.
- (b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where x_{rel} is the excursion of the block away from the point $\frac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\frac{\mu_k mg}{k}$.

- (d) The amplitude of its motion is its original displacement, $A = \frac{\mu_s mg}{k} - \frac{\mu_k mg}{k}$. It first comes to rest at spring extension $\frac{\mu_k mg}{k} - A = \frac{(2\mu_k - \mu_s)mg}{k}$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

- (c) The graph of the motion looks like this:

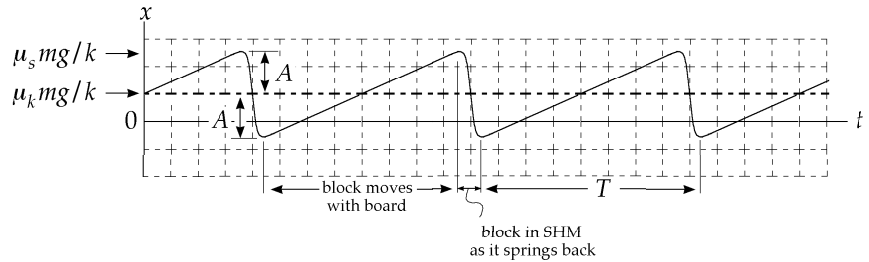


FIG. P15.74(c)

- (e) The time during each cycle when the block is moving with the board is $\frac{2A}{v} = \frac{2(\mu_s - \mu_k)mg}{kv}$. The time for which the block is springing back is one half a cycle of simple harmonic motion, $\frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \pi\sqrt{\frac{m}{k}}$. We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v . Since v is small compared to $\frac{2A}{\pi\sqrt{\frac{m}{k}}}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}.$$

continued on next page

$$(f) \quad T = \frac{2(0.4 - 0.25)(0.3 \text{ kg})(9.8 \text{ m/s}^2)}{(0.024 \text{ m/s})(12 \text{ N/m})} + \pi \sqrt{\frac{0.3 \text{ kg}}{12 \text{ N/m}}} = 3.06 \text{ s} + 0.497 \text{ s} = 3.56 \text{ s}$$

$$\text{Then} \quad f = \frac{1}{T} = \boxed{0.281 \text{ Hz}}.$$

$$(g) \quad T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi \sqrt{\frac{m}{k}} \text{ increases as } m \text{ increases, so the frequency } \boxed{\text{decreases}}.$$

$$(h) \quad \text{As } k \text{ increases, } T \text{ decreases and } f \boxed{\text{increases}}.$$

$$(i) \quad \text{As } v \text{ increases, } T \text{ decreases and } f \boxed{\text{increases}}.$$

$$(j) \quad \text{As } (\mu_s - \mu_k) \text{ increases, } T \text{ increases and } f \boxed{\text{decreases}}.$$

***P15.75** (a) Newton's law of universal gravitation is

$$F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2} \left(\frac{4}{3} \pi r^3 \right) \rho$$

Thus,

$$F = -\left(\frac{4}{3} \pi \rho G m \right) r$$

Which is of Hooke's law form with

$$k = \frac{4}{3} \pi \rho G m$$

$$(b) \quad \text{The sack of mail moves without friction according to} \quad -\left(\frac{4}{3} \right) \pi \rho G m r = ma$$

$$a = -\left(\frac{4}{3} \right) \pi \rho G r = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$$\omega = \sqrt{\frac{4\pi\rho G}{3}} \quad \text{and period} \quad T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$$

$$\text{The time for a one-way trip through the earth is} \quad \frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$$

We have also

$$g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3 \rho}{3R_e^2} = \frac{4}{3} \pi \rho G R_e$$

$$\text{so} \quad \frac{4\rho G}{3} = \frac{g}{(\pi R_e)} \quad \text{and} \quad \frac{T}{2} = \pi \sqrt{\frac{R_e}{g}} = \pi \sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 2.53 \times 10^3 \text{ s} = \boxed{42.2 \text{ min}}.$$

ANSWERS TO EVEN PROBLEMS

P15.2 (a) 4.33 cm; (b) -5.00 cm/s;
(c) -17.3 cm/s²; (d) 3.14 s; 5.00 cm

P15.6 see the solution

P15.8 12.0 Hz

P15.4 (a) 15.8 cm; (b) -15.9 cm;
(c) see the solution; (d) 51.1 m; (e) 50.7 m

P15.10 18.8 m/s; 7.11 km/s²

472 Oscillatory Motion

- P15.12** (a) 1.26 s; (b) 0.150 m/s; 0.750 m/s²;
 (c) $x = -3 \text{ cm} \cos 5t$; $v = \left(\frac{15 \text{ cm}}{\text{s}}\right) \sin 5t$;
 $a = \left(\frac{75 \text{ cm}}{\text{s}^2}\right) \cos 5t$
- P15.14** (a) $\frac{v}{\omega}$; (b) $x = -\left(\frac{v}{\omega}\right) \sin \omega t$
- P15.16** (a) 126 N/m; (b) 0.178 m
- P15.18** (a) 0.153 J; (b) 0.784 m/s; (c) 17.5 m/s²
- P15.20** (a) 100 N/m; (b) 1.13 Hz;
 (c) 1.41 m/s at $x = 0$;
 (d) 10.0 m/s² at $x = \pm A$; (e) 2.00 J;
 (f) 1.33 m/s; (g) 3.33 m/s²
- P15.22** (a) 1.50 s; (b) 73.4 N/m;
 (c) 19.7 m below the bridge; (d) 1.06 rad/s;
 (e) 2.01 s; (f) 3.50 s
- P15.24** (a) 0.218 s and 1.09 s; (b) 14.6 mW
- P15.26** The position of the piston is given by
 $x = A \cos \omega t$.
- P15.28** $\frac{g_C}{g_T} = 1.0015$
- P15.30** 1.42 s; 0.499 m
- P15.32** (a) 3.65 s; (b) 6.41 s; (c) 4.24 s
- P15.34** (a) see the solution;
 (b), (c) 9.85 m/s²; agreeing with the
 accepted value within 0.5%
- P15.36** (a) 2.09 s; (b) 4.08%
- P15.38** 203 $\mu\text{N} \cdot \text{m}$
- P15.40** see the solution
- P15.42** see the solution
- P15.44** (a) 2.95 Hz; (b) 2.85 cm
- P15.46** see the solution
- P15.48** either 1.31 Hz or 0.641 Hz
- P15.50** 1.56 cm
- P15.52** (a) 0.500 m/s; (b) 8.56 cm
- P15.54** $A = \frac{\mu_s g}{4\pi^2 f^2}$
- P15.56** see the solution
- P15.58** (a) $k = \frac{4\pi^2 m}{T^2}$; (b) $m' = m \left(\frac{T'}{T}\right)^2$
- P15.60** (a) $x = (-2 \text{ m}) \sin(10t)$; (b) at $x \pm 1.73 \text{ m}$;
 (c) 98.0 mm; (d) 52.4 ms
- P15.62** (a) decreased by 0.735 m;
 (b) increased by 0.730 s;
 (c) decreased by 120 J; (d) see the solution
- P15.64** (a) 3.56 Hz; (b) 2.79 Hz; (c) 2.10 Hz
- P15.66** (a) $\frac{1}{2} \left(M + \frac{m}{3}\right) v^2$; (b) $T = 2\pi \sqrt{\frac{M + \frac{m}{3}}{k}}$
- P15.68** see the solution; (a) $k = 1.74 \text{ N/m} \pm 6\%$;
 (b) 1.82 N/m $\pm 3\%$; they agree;
 (c) 8 g $\pm 12\%$; it agrees
- P15.70** (a) 5.20 s; (b) 2.60 s; (c) see the solution
- P15.72** see the solution; $T = \left(\frac{2}{r}\right) \sqrt{\frac{\pi M}{\rho g}}$
- P15.74** see the solution; (f) 0.281 Hz;
 (g) decreases; (h) increases; (i) increases;
 (j) decreases